Setting $\gamma = 0$, Eq. (11) yields the solution to special case (1).[3] Setting both $\gamma = 0$ and $\lambda = 0$, Eq. (11) yields the solution to special case (2).[4] Setting $\gamma = 0$ and letting $\lambda \to \infty$, Eq. (11) yields the solution to special case (3).[5] Finally, letting both $\gamma \to \infty$ and $\lambda \to \infty$, Eq. (11) yields the solution to special case (4). In this last case the resulting expression for the deflection becomes

$$w = \frac{4qa^4}{D\pi^5} \sum_{m=1,3,5}^{\infty} \frac{1}{m^5} \left\{ 1 - \left[\frac{(\alpha_m \cosh\alpha_m + \sinh\alpha_m) \cosh (m\pi y/a) - \sinh\alpha_m (m\pi y/a) \sinh(m\pi y/a)}{\cosh\alpha_m \sinh\alpha_m + \alpha_m} \right] \right\} \sin\frac{m\pi x}{a} \quad (12)$$

which is simpler than that which is offered in Timoshenko's text[6] where the total deflection is given as a superposition of two separate solutions. However, after a certain amount of algebraical work, it has been shown that Timoshenko's result could be simplified to agree with Eq. (12).

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Prediction of Adiabatic Wall Temperatures in Film-Cooling Systems

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Nomenclature

u_C = velocity of coolant fluid emerging from slot, presumed uniform across slot
u_G = velocity of mainstream, presumed uniform along plate
x = distance along plate measured from slot
X = dimensionless distance defined by Eq. (9)
y_C = width of slot in direction normal to wall
ϵ = effectiveness of film cooling, i.e., temperature difference between the adiabatic wall and mainstream, divided by temperature difference between coolant in the slot and mainstream
ν = kinematic viscosity of fluid, presumed uniform

1. Introduction

In a recent paper in this journal, Librizzi and Cresci[1] described a simple theory, based on a suggestion by Libby, for the calculation of the adiabatic wall temperature produced by film cooling. The theory starts from the assumption that the coolant fluid is fully mixed with the material in the boundary layer, which, therefore, exhibits a uniform temperature along a normal to the wall; a discontinuity of temperature must appear at the outer "edge" of the boundary layer. The growth of the boundary layer is supposed to follow the usual laws, without any special influence of injection. The authors successfully compared the predictions of the theory with data published by Nishiwaki et al.[2]

The purpose of the present note is threefold: 1) to mention some independent developments of the same theory, 2) to report on the successes and failures achieved with it, and 3) to mention some later methods of predicting the thermal effects of film cooling. The writer believes that the conceptual simplicity of the theory of Ref. 1 renders it a valuable starting point for understanding film cooling; however, its limitations have to be recognized.

In order to emphasize the similarities in the theories, discussion is here restricted to two-dimensional uniform-property flows.

2. Earlier Theories

Stollery[3] and Kutateladze and Leont'ev[4] have developed theories that are almost identical in starting points and results with those of Ref. 1. The similarities may be seen by expressing the results of each author in the form of an equation for the film-cooling effectiveness ϵ in terms of the Reynolds number of the coolant fluid at the slot exit $u_C y_C/\nu$, the ratio of coolant velocity to mainstream velocity u_C/u_G, and the ratio of downstream distance to slot width x/y_C.

Stollery deduced

$$\epsilon = 3.09 \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{-0.8} \left(\frac{u_C y_C}{\nu}\right)^{0.2} \quad (1)$$

Kutateladze and Leont'ev deduced

$$\epsilon = 3.1 \left\{4.16 + \frac{u_G}{u_C}\frac{x}{y_G} \left(\frac{u_C y_C}{\nu}\right)^{-0.25}\right\}^{-0.8} \quad (2)$$

Librizzi and Cresci deduced

$$\epsilon = 3.0 \left\{3.0 + \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{0.8} \left(\frac{u_C y_C}{\nu}\right)^{-0.2}\right\}^{-1} \quad (3)$$

It is easily seen that, when x/y_C is large, the three expressions lead to almost identical expressions for ϵ. The differences derive from slightly divergent treatments of the upstream region and from trivial differences in the multiplying constant. Incidentally, Stollery has pointed out in another report[5] that similar formulas are implicit in the works of earlier authors, namely, those of the following:

Wieghardt[6] (together with the assumption that the boundary-layer thickness equals $0.37x(ux/\nu)^{-1.2}$)

$$\epsilon = 5.44 \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{-0.8} \left(\frac{u_C y_C}{\nu}\right)^{0.2} \quad (4)$$

Hartnett et al.[7]

$$\epsilon = 3.39 \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{-0.8} \left(\frac{u_C y_C}{\nu}\right)^{0.2} \quad (5)$$

Tribus and Klein[8]

$$\epsilon = 4.62 \left(\frac{u_G}{u_C}\frac{x}{y_C}\right)^{-0.8} \left(\frac{u_C y_C}{\nu}\right)^{0.2} \quad (6)$$

The differences in the coefficient derive from different assumptions about the temperature profile, i.e., about the extent to which the coolant fluid is fully mixed with the fluid in the boundary layer. The better the mixing, the lower the coefficient; this is why the theory of Libby-Stollery-Kutateladze, which assumes complete mixing, gives the lowest coefficient of all.

3. Comparison with Experiment

Kutateladze and Leont'ev compared their formula with the same data as were selected by Librizzi and Cresci, namely

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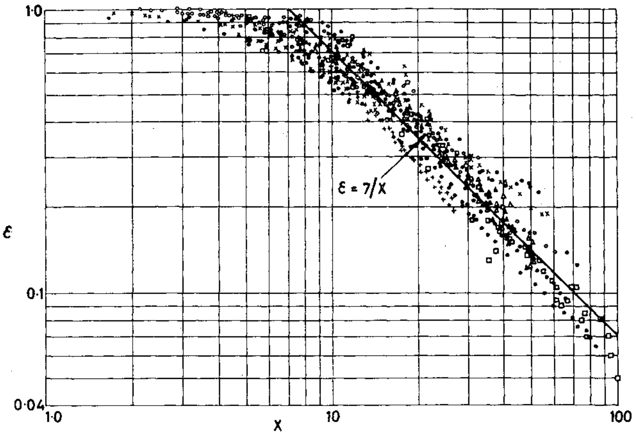

Fig. 1 Agreement between Eq. (9) and the data of Refs. 6, 7, and 10–16. □ = μ_C/μ_G <1 **(Ref. 6);** ○ = μ_C/μ_G >1, <1 **(Ref. 10);** × = μ_C/μ_G >1, <1 **(Ref. 11);** △ = μ_C/μ_G <1 **(Ref. 7);** + = μ_C/μ_G >1 **(Ref. 13);** ● = μ_C/μ_G <1 **(Refs. 14 and 15).**

those of Nishiwaki, Hirata and Tsuchida.[2] Of course they found the same satisfactory agreement. However, there is a definite tendency for the experimental effectivenesses to exceed the predicted ones at large distances from the slot; this suggests that the coefficient should indeed be greater than 3.1.

The Stollery formula was compared by Cole and Peerless[9] with experimental data of Wieghardt,[6] Chin et al.,[10, 11] Hatch and Papell,[12] Hartnett et al.,[7] Seban and Back,[13, 14] Seban,[15] and Papell and Trout.[16] Fairly good agreement was found between predictions and measurements in view of the wide range of geometries which had been used, but only when the coolant was injected with a velocity lower than that of the mainstream ($u_C < u_G$). For experiments in which the injection velocity was appreciably greater than that of the mainstream, Cole and Peerless found better agreement with a correlation suggested by the present writer;[17] this was based on the assumption that the flow near the wall behaved more like a jet than a boundary layer, and implied

$$\epsilon = 3.4 \left\{ \frac{x}{y_C} \left| 1 - \frac{u_G}{u_C} \right| \right\}^{0.5} \tag{7}$$

This formula differs from those already mentioned in several respects, including the different exponent of x/y_C and the absence of a viscosity term. Although successful for the case $u_C > u_G$, this formula failed to represent the data for $u_C < u_G$. Indeed, even for $u_C > u_G$ departures from the prediction became noticeable at large x/y_C, where the slope of the curve tended to correspond more with the exponent -0.8 than with -0.5. Obviously a flow that is jet-like near the slot reverts to the boundary-layer type farther downstream.

4. Further Developments

The experiences just mentioned led Spalding, Jain, and Nicoll[18] to propose an artificially contrived formula that would fit all of the data, regardless of the value of u_C/u_G. Their result can be expressed, for uniform properties, as

$$\begin{aligned} &\text{for } X < 7: \quad \epsilon = 1 \\ &\text{for } X \geqslant 7: \quad \epsilon = 7/X \end{aligned} \tag{8}$$

where

$$X \equiv 0.91 \left(\frac{u_G x}{u_C y_C} \right)^{0.8} \left(\frac{u_C y_C}{\nu} \right)^{-0.2} + 1.41 \left\{ \left| 1 - \frac{u_G}{u_C} \right| \cdot \frac{x}{y_C} \right\}^{0.5} \tag{9}$$

This formula represents a fairly obvious combination of Eqs. (1) and (7), together with some nonsystematic adjustment of constants to improve the agreement with experiment.

Figure 1 displays the agreement between this formula and the data of Refs. 6, 7, and 10–16. Although the scatter could certainly be reduced by a better selection of constants, Eq. (9) presumably represents nearly as good a simple correlation formula as can be derived. It must be remembered that the plotted data correspond to values of u_C/u_G ranging from nearly zero up to about 14, and that the geometrical arrangements at the slot were not identical in shape.

Efforts to improve the correlation formula have not been continued by the present writer and his associates, because there seem to be good prospects of developing a more satisfactory (but more complex) theory of the development of turbulent boundary layers which takes account of the way in which the entrainment of fluid from the mainstream into the boundary layer is influenced by the velocity profile in this layer. Preliminary reports of the new theory and its problems have been written.[19, 20] A central concept in the new theory is that of the amount of fluid flowing in the boundary layer; the theory can, therefore, be regarded as a sophisticated version of the basic ideas reported in Refs. 1, 3, and 4.

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Boundary-Layer Control for Increasing Lift by Blowing

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Nomenclature

x	= distance along the wing section surface
y	= distance from the wing section surface
c	= wing chord
s	= slot width
Re	= Reynolds number ($= U_\infty c/\nu$)
ρ	= density of air
v_j	= velocity of the jet
U_∞	= freestream velocity
$U(x)$	= velocity distribution over the wing section
$\bar{U}_K$	= mean value of the velocity on the deflected flap
$u(y)$	= velocity distribution in the boundary layer
ϑ	= momentum loss thickness $= \int_0^\infty \rho u(y)/\rho_\infty U_\infty [1 - u(y)/U_\infty]dy$
c_μ	= momentum coefficient $= 2[(\rho_j v_j^2 s)/(\rho_\infty U_\infty^2 c)]$
$c_{\mu S}$	= minimum momentum coefficient required for preventing boundary-layer separation
TE	= trailing edge
S	= point of separation

It is well known that the actual lift of a wing with a deflected trailing edge flap is far below the lift predicted by potential flow theory because of boundary-layer separation. This separation can be prevented by blowing a high velocity air jet out of a narrow slot near the flap knee into the boundary layer (see Fig. 1a). The minimum momentum coefficient required for avoiding boundary-layer separation and for thus attaining the theoretical lift has so far only been found by experiments.[1] In the following, a simple method will be described for calculating the momentum coefficient necessary to prevent separation. This is achieved by boundary-layer calculations, using some empirical results from detailed measurements on boundary layers with blowing.

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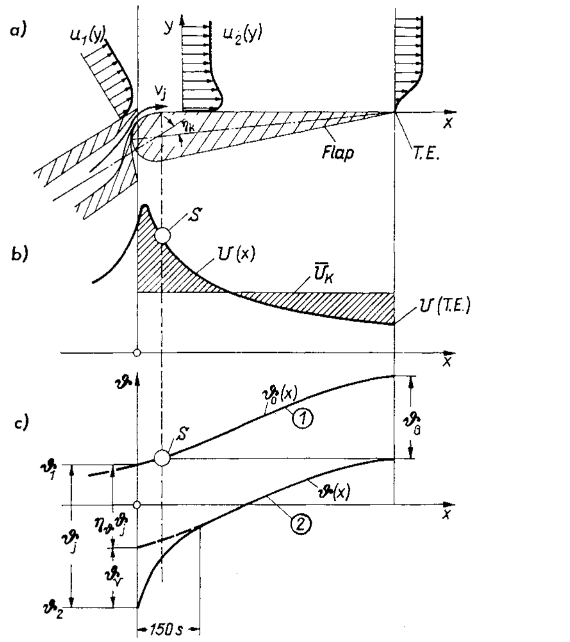

Fig. 1 Boundary-layer control by blowing over a deflected trailing edge flap. a) Boundary-layer profiles on the flap. b) Velocity distribution on the flap from potential flow theory. c) Distribution of momentum loss thickness along the deflected flap (① without blowing; ② with blowing).

At the point where the jet is blown into the boundary layer, the momentum loss thickness of the boundary-layer ϑ changes abruptly, as is indicated in Fig. 1c. This jump in momentum loss thickness is

$$\vartheta_j = \vartheta_2 - \vartheta_1 \tag{1}$$

where ϑ_1 and ϑ_2 are the momentum loss thicknesses just in front of the slot and behind the slot, respectively.

From simple momentum considerations, the following relation between c_μ and ϑ_j is found[2]:

$$\vartheta_j = -\tfrac{1}{2}c_\mu c\,[1 - (U_\infty/v_j)] \tag{2}$$

One part ϑ_v of the "equivalent momentum thickness ϑ_j" of the jet is lost by friction and mixing and only the remaining part $\eta_\vartheta \vartheta_j$ is actually available for boundary-layer control. In order to get information on these losses, detailed boundary-layer measurements were carried out for different jet velocities v_j and different constant mainstream velocities U_∞.[2,3] In all cases within the range $v_j/U_\infty \geq 2$, which was covered by these tests, the momentum thickness $\vartheta(x)$ followed the pattern shown in Fig. 1c. That is, behind the jump of the amount ϑ_j at the slot, the momentum thickness rises very steeply within a distance of about 150 s from the slot. It then follows a curve that is parallel to the curve $\vartheta_0(x)$ of the corresponding case without blowing. From these measurements the following general law was found for the "efficiency factor" η_ϑ of the jet:

$$\eta_\vartheta = 0{,}85\,[1 - (U_\infty/v_j)] \tag{3}$$

In the procedure utilized to find the theoretical estimation of the minimum momentum coefficient required for preventing boundary-layer separation, the next step is the calculation of the potential flow velocity distribution of the wing section with the deflected flap and, thus, with a finite suction peak at the flap knee.[4] By applying boundary-layer calculations to this velocity distribution,[5] the separation point S in the case of no blowing can easily be found (see Fig. 1b). In order to obtain the lift predicted by potential flow theory, the separation point has to be shifted to the trailing edge by blow-